Archimedes’ Principle

Purpose
To connect the words of Archimedes’ Principle to the actual behavior of submerged objects.
To examine the cause of buoyancy, that is the variation of pressure with depth in a fluid.
To use Archimedes’ Principle to determine the density of an unknown material.

Explore the Apparatus
We’ll use the Buoyancy Apparatus in this lab activity. You can get quick access to help by rolling your mouse over most objects on the screen.

Figure 1 – The Buoyancy Apparatus
When working with buoyant forces we need to be able to measure the weight and volume of the (fully or partially) immersed objects and the fluid (water in this lab) displaced. A hanging scale and a digital scale are available for measuring weights.
The overflow of water from the tank spills into a graduated cylinder. The volume of water displaced can be read from the graduated cylinder; and the weight of the fluid can be found by weighing the graduated cylinder before and after overflow.

Two simple and two compound objects are available for study. They are composed of cork, aluminum, and an unknown material. These objects can be weighed in air and when partially or fully submerged. The hanging scale is raised and lowered by dragging the metal arm that supports it. When an object begins to enter the water, the volume of water it displaces overflows into the graduated cylinder. The weight of the water displaced can be found by weighing the cylinder before and after it’s filled with overflow water. Both the hanging scale and the graduated cylinder can be read more precisely by zooming in. The graduated cylinder is emptied by releasing it above the tank.

You’ll find that some actions that could be done with real apparatus are forbidden. For example, if you removed the cork and aluminum object from the hanging scale and then attached a new object to the scale, you’d find no further overflow is possible without first emptying the cylinder. Similarly, you can’t weight more than one object on the digital scale at a time.
And you can’t make a mess by lowering an object into the water while the cylinder is on the digital scale. The rod holding the hanging scale just won’t move.

Another thing that is largely forbidden is dragging the screen when you’re zoomed in.

Just as with the principle we’re studying, many of the actions just mentioned are hard to picture without direct experience. Let’s take care of that now. First let’s remind ourselves or the principle we’re investigating.

**Archimedes’ Principle:** A body wholly or partially immersed in a fluid will experience a buoyant force equal to the weight of the fluid displaced.

Now for some phrases that we need to experience directly.

1. **“measure the weight and volume of the (fully or partially) immersed objects”**
2. **“buoyant forces”**

Let’s try it with the system in Figure 1. Notice that the hanging scale reads zero initially. Right (or CTRL) click and zoom in on the scale. It’s close to zero. This sort of scale is not very precise or accurate. Our simulation follows that tradition. (Zoom back out.)

Now drag the cork and aluminum compound object and drop it somewhere below the hanger. You should find that it weighs about 4.4 N. Drag and drop it on the digital scale and you’ll get a more precise answer of about 4.41 N. Which should you use? This is an excellent example of where significant digits come into play. As you know from Newton’s Law of Gravitation, object weigh a bit more close to Earth. Is that what we’re seeing here? Of course not. We just have one device that’s more precise than the other. When you use both tools in one experiment, just be sure to defer to the least precise tool when doing calculations. E.g., 4.41 N – 4.4 N = 0.0 N, not 0.01 N since we can’t assume that the 4.4 N value is actually 4.40 N. So we drop the more precise .01 from the more precise value to make it 4.4 N – 4.4 N.

Now for the “weight when fully or partially immersed in the water.” Drag the object back to the hanging scale. Slowly drag the support arm above the hanging scale until the aluminum is about half submerged. The scale might read about 3.7 N. That’s the “weight when…” We sometimes refer to this as the **apparent weight**.

So what’s the buoyant force? Right now it’s about .7 N. The buoyant force is what the scale no longer has to support because a new upward force is present. It’s an upward force caused by the pressure on the bottom of the disk. If you lower it more, the buoyant force becomes larger and the apparent weight is reduced by an equal amount.

3. **How about “weight of the fluid displaced”?**

The buoyant force is not just the missing upward force. It’s also equal to the weight of the fluid displaced into the graduated cylinder. Drag the cylinder onto the digital scale. It might read about 1.700 N. That’s the weight of the fluid plus the weight of the cylinder. So we need to subtract the weight of the cylinder. So we need to empty the water.

Try dragging and releasing the cylinder somewhere above the tank. No dice. You have to raise the scale first. Now try it. Once it’s empty, return it to the digital scale. If you subtract its weight from the previous total you’ll get about the same value as the apparent change in weight you go before! Got it?

4. **How about “volume of the fluid displaced”?**

Displace some more water into the graduated cylinder. Right click and zoom in on the cylinder. You can now read the volume in ml.

5. **It’s important to note that when we say that the upward buoyant force equals the downward weight we’re really talking about magnitudes.** We’ll use free body diagrams to visualize these forces, but we’ll use the scalar quantities when we write our equations.
Theory

Archimedes’ Principle

Archimedes’ Principle is very easy to state in words, but learning how to use it takes practice in a variety of situations. In this lab we’ll first explore the literal meaning of this principle. Then we’ll put it to use in a couple of situations. You’ll use the wording of the principle to draw free-body diagrams (FBDs) and create equations that can be used to find unknown quantities such as the volume, mass or density of an object or of the fluid it’s immersed in.

<table>
<thead>
<tr>
<th>Archimedes’ Principle:</th>
<th>A body wholly or partially immersed in a fluid will experience a buoyant force equal to the weight of the fluid displaced.</th>
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</table>

Without a bit more information, this is little more than magical incantation! Just what is the source of this buoyant force?

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Figure 2 – Pressure vs. Depth in a Fluid

In Figure 2 we see a solid cylinder being gradually lowered into a tank of fluid. Let’s assume that the solid cylinder is wood and the fluid is water. The gauge pressure at any point a distance \( h \) below the top surface of the water equals \( \rho_{\text{water}} g h \). This pressure acts in all directions but we are only concerned with the pressure at the top and bottom of the wooden cylinder. We can ignore horizontal pressures since at any depth the pressure is the same, so each side of the cylinder will experience the same pressure. Thus there will be no net horizontal pressure.

In Figure 2a the cylinder is surrounded by air on all sides. The air pressure on the bottom will be slightly greater than that on the top, but it would be insignificant. We’ll ignore this slight effect.

In Figure 2b, the bottom of the cylinder is submerged.

The downward pressure on the top is again atmospheric, \( P_{\text{atm}} \).

The upward pressure on the bottom is \( P_{\text{atm}} + \) the gauge pressure, \( \rho g h \), where \( h \) is the depth of the water at the bottom of the cylinder.
Replacing “top” and “bottom” with the more general “1” and “2,” the pressure difference between the top and bottom is

$$\Delta P = P_2 - P_1$$

(Note: $P_2$ and $P_1$ refer to $P_{up}$ and $P_{down}$ respectively.)

$$P_2 = \rho_w g h_2$$  \hspace{1cm} \text{Equation 1}

As a result there is an upward buoyant force, $F_{buoy}$, equal to this pressure times the top surface area.

$$F_{buoy} = P_2 A$$  \hspace{1cm} \text{Equation 2}

In Figure 2c, $P_2$ is larger due to the greater depth, thus $F_{buoy}$ is proportionally larger.

This buoyant force will continue to increase as more of the cylinder is submerged until Figure 2d where the maximum $\Delta P$ is achieved. In Figure 2e, $P_2$ is larger than in 2d, but there is now a $P_1$. Notice the $\Delta P$ arrow in 2d through 2f. It remains the same because each increase in $P_2$ is matched by an equal increase in $P_1$. So a more general statement than Equation 2 which works in each part of Figure 2 is

$$F_{buoy} = \Delta P A$$

$$F_{buoy} = (P_2 - P_1) A$$  \hspace{1cm} \text{Equation 3}

The origin of the buoyant force should make sense now, but what about the “weight of the fluid displaced?”

The weight of the fluid now occupied by the cylinder = $m_{cylinder} g$.

Using $\rho = \text{mass/volume}$ we can replace the mass with $\rho \times \text{volume}$.

And the volume of the cylinder is the area of its base times its height. Hence,

Weight of cylinder = weight of fluid displaced = $\rho g V = \rho g (h_2 - h_1) A$

which, by Equation 3, equals the buoyant force!

This actually applies to objects of any shape but we’ll omit that derivation.

So the buoyant force makes sense in terms of the difference in pressure and area. And further examination tells us that this same ($\Delta P \cdot A$) equals the weight of the fluid displaced.

We can also understand it from another direction by realizing that when water fills the same volume, it must have an upward force equal to its weight. Otherwise it would not stay at rest. Any other object inserted in the same volume would experience the same force because the same pressures would be present.
**Archimedes’ Principle with additional applied forces.**

Most of our work with Archimedes’ principle includes additional forces. For example,

- The submerged body itself has a weight which opposes the buoyant force.
- A string might be pulling upward or downward on a submerged body. We’ll refer to the force it exerts as the apparent or submerged weight, W’.
- The buoyant force might be used to partially or fully support another object. For example, a life jacket has to supply enough buoyant force to support itself as well as the person wearing it.

These situations are generally dealt with using the tools of translational equilibrium. That is, if a body is in equilibrium, the sum of the forces in any direction will equal zero. A buoyant force is just another force.

Just as important, a careful free-body diagram, FBD, is essential in working out what object(s) are involved as well as all forces involved.

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**Example 1.** A steel anchor weighs 2000 N in air. How much force would be required to support it when submerged in water? That is, what is its apparent weight, W’, when submerged? W’ is an upward support force in this case.

There is an upward support force, W’, an upward buoyant force, $F_b$, and the downward gravitational force, that is, its weight, W.

The sum of these forces equals zero. So, (using magnitudes) $F_b + W' = W$

Other considerations:

$F_b = \text{weight of fluid (water) displaced}

= \rho_{\text{water}} g V_{\text{water}} = \rho_{\text{water}} g V_{\text{anchor}}$ since the anchor displaces an equal volume of water.

And since we know the weight and mass of the anchor we can find its volume by looking up its density and using $\rho_{\text{steel}} = \frac{m_{\text{anchor}}}{V_{\text{anchor}}}$

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**Procedure**

In the following sections you’ll collect and analyze data to investigate Archimedes’ Principle in some typical situations such as the one suggested in example 1. You won’t be provided step by step directions but you’ll have blanks provided for the data you need to collect. This method is being used because buoyancy problems are always sort of free form. It’s not about finding an equation that fits. Rather you try to find a way that your information fits into the Archimedes’ Principle.

You’ll also get guidance in developing FBDs as well as equations describing your FBDs. Note that the equations are in scalar form. That is, we’ll say

“The sum of the upward forces = the sum of the downward forces” (all positive magnitudes)

rather than

“The sum of the upward forces + the sum of the downward forces = 0.” (all signed vectors)
I. Confirm Archimedes’ Principal for $\rho_{\text{object}} > \rho_{\text{liquid}}$ using overflow

In this section we’ll directly measure the buoyant force and the weight of the water displaced and confirm that they are equal. Our object will be an aluminum disk.

Weighing the aluminum disk in air (Fig. 6a) gives us $W$. Weighing it when submerged (6c) gives us $W'$. From our FBD (6e) we can see that

$$F_{\text{buoyant}} + W' = W, \text{ so } F_b = W - W' \quad \text{Equation 4}$$

Weighing the empty graduated cylinder (6b) gives us $W_{\text{cyl}}$. Weighing the cylinder after receiving the overflow water (6d) gives us $W_{\text{cyl}} + \text{water displaced}$. Logically,

$$F_b = W_{\text{water displaced}} = (W_{\text{cyl}} + \text{water displaced}) - W_{\text{cyl}} \quad \text{Equation 5}$$

1. Using your apparatus, collect and record the data below.

   $W_d =$ ___________ N  
   $W_{\text{cyl}} =$ ___________ N  
   $W_d' =$ ___________ N  
   $W_{\text{cyl}} + \text{water} =$ ___________ N  

2. Calculate the buoyant force, $F_b$, twice, using Equation 4, and Equation 5.

   $F_b =$ ___________ N  (Equation 4)  
   $F_b =$ ___________ N  (Equation 5)  

   % difference = ________ %  

Show your calculations of $F_b$ for each method and the percentage difference.
**II. Confirm Archimedes’ Principle for \( \rho_{\text{object}} < \rho_{\text{liquid}} \) using overflow**

In this section we’ll directly measure the buoyant force and the weight of the water displaced, and confirm that they are equal. Our object will be a cork disk. This time you fill in all the missing pieces. Sort of like the kids menu at Waffle House. Use the previous section as a model. Collect and record the data required below.

→→ Also fill in the missing pieces below including Figures 7a, c, and d, FBD 7e, and Equation 6 ←←

![Figure 7a](image1.png) ![7b](image2.png) ![7c](image3.png) ![7d](image4.png)

Weighing the cork disk in air (Fig. 7a) gives us \( W_{\text{cork}} \). Weighing it when partially submerged (7c) gives us \( W_{\text{cork}}' \).

But it floats so \( W_{\text{cork}}' = \text{________ N} \)

From FBD 7e we can say that

\[
W_{\text{cork}} = \text{________ N}
\]

Weighing the empty graduated cylinder (7b) gives us \( W_{\text{cyl}} \). Weighing the cylinder after receiving the overflow water (7d) gives us \( W_{\text{cyl} + \text{water displaced}} \). Logically,

\[
W_{\text{water displaced}} = (W_{\text{cyl + water displaced}}) - W_{\text{cyl}}
\]

\[
W_{\text{cyl + water}} = \text{________ N}
\]

2. Calculate the buoyant force, \( F_b \), twice, using Equation 6, and Equation 7.

\[
F_b = \text{________ N} \quad \text{(Equation 6)}
\]

\[
F_b = \text{________ N} \quad \text{(Equation 7)}
\]

% difference = \text{________ %}

Show your calculations of \( F_b \) for each method and the percentage difference.
**III. Determine the density of a unknown material.**

We’ve confirmed that Archimedes’ Principle correctly describes the forces acting on a body wholly or partially submerged. That gives us a starting point for more complex situations that are not so straight-forward. One of your object choices is a compound object with a cork on top and a rod of negligible volume connecting it to a black unknown object below it. The unknown object is more dense than cork; otherwise the system would try to flip upside-down. Your task is to determine the density of the unknown object. In the process you’ll first have to determine the density of the cork.

**Density of the cork**

You already know the weight of the cork from part B. From the weight you can calculate the mass. What you need is the cork’s volume. You can find that using the other compound object – the cork with (shiny) aluminum attached below it.

1. Attach the cork/aluminum object to the hanging scale. Lower the system slowly watching the water overflow into the cylinder until the aluminum cylinder becomes completely submerged. Zoom in a few times on the graduated cylinder to read and record the volume of water displaced by the aluminum.

   **NOTE:** The graduated cylinder measures volume in ml. You’ll eventually convert it to m³ in the data table. Be sure to think about this conversion. A milliliter is 1 cm³. Careful.

Continue a bit further to see that the connecting rod is ignored, causing no overflow. Continue on and notice that the cork becomes completely submerged. You want to know the volume of the cork. Fill in the data table below to find the volume of the cork, convert it to m³, and compute its density in kg/m³.

| Volume_{al} = | ml | Volume_{al+cork} = | ml | Volume_{cork} = | ml | Volume_{cork} = | \times 10^{-4} m^3 | Weight_{cork} = | N (from part B) | Mass_{cork} = | kg | Density_{cork} = | kg/m^3 |
|---------------|----|-------------------|----|----------------|----|--------------------|-----------------|----------------|---------------|-------|----------------|-------|

   Show your calculations of items 3-8 here.

**Density of the unknown material**

Return the cork/aluminum object to its home. Empty the graduated cylinder and return it home.

2. Attach the unknown system to the hanging scale. Gradually lower it as far down as it will go. What is different about this compound system relative to the cork/aluminum system?
3. Draw a free-body diagram in the space to the right. You will need separate weight vector for each object. Use $W_u$ for the weight of the unknown material. And you’ll need a single $F_{buoyant}$ vector for the pair. There will be no apparent weight vectors since the system floats. That is, it appears to be weightless.

From our FBD we can say that

$$F_{buoyant} = W_c + W_{unknown} \tag{Equation 8}$$

Now what? We want to know the density of the unknown material, $\rho_u$. If we could find its weight we could use

$$W_u = \rho_u g V_u.$$  
We know $g$ and we can measure $V_u$ using our overflow system. So we just need its weight, $W_u$.

We also know $W_c$, so if we knew $F_{buoyant}$ we’d have it.

Have a look at your system. How is the $F_{buoyant}$ related to the water in your graduated cylinder? How is it related to the scale reading before and after you lower the objects into the water? Either can be used to determine $F_b$.

4. We know that $F_b = W_c + W_u$, and $W_u = \rho_u g V_u$.

Combine the two equations and solve for $\rho_u$. Show you algebra here.

5. Determine the total buoyant force, $F_b$ by either Equation 8 or from the weight of the water in the overflow graduated cylinder. Record $F_b$ in the table.

<table>
<thead>
<tr>
<th>Weight of cork</th>
<th></th>
<th>Show your calculations of items 2 and 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{cork}$</td>
<td>N</td>
<td>(from part B)</td>
</tr>
<tr>
<td>$F_b$</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>kg/m$^3$</td>
<td></td>
</tr>
</tbody>
</table>

The unknown material is concrete. Check on line to see if your result is reasonable. Note that concrete varies in density depending on its manufacture.