Dynamics – Pulleys, Ramps, and Friction

Purpose

To investigate the vector nature of forces.  
To practice the use free-body diagrams (FBDs).  
To learn to apply Newton’s Second Law to systems of masses connected by pulleys.  
To develop a clear concept of the idea of apparent weight.  
To explore the distinction between static and kinetic friction.  
To explore the behavior of carts on ramps with and without friction.

Equipment

Virtual Dynamics Track      PENCIL      Graphing Software (e.g., Logger Pro)

Explore the Apparatus

Open the Virtual Dynamics Lab on the website. You should see the low-friction track and cart at the top of the screen. At the bottom you’ll see a roll of massless string, several masses and a mass hanger. Roll your pointer over each of these to view the behavior of each. Note the values of the masses. Also note that the empty hanger’s mass is 50g. If you drag over the cart you’ll see that its mass is 250 g.

Let’s take a trial run with the apparatus. Remember, everyone needs to take a turn at this.

- Turn on the cart brake (stop sign) and then move the cart to the middle of the track.
- Drag the roll of massless red string up to the cart and release it with your pointer (mouse button up) somewhere just to the right of the cart. (Fig. 1a) You should see a short segment of string connecting the cart to the pointer. (Fig. 1b) Without pressing any mouse buttons, move your pointer to the right. The string will follow.

Continue until your pointer passes the pulley by a bit. (Fig. 1c) Now move downward. When the scissors appear, click with your mouse. (Fig. 1d) The string will extend downward and a loop will appear at the end. (Fig. 1e)

- Drag the mass hanger until its curved handle is a bit above the loop and release (Fig. 1f). The hanger will attach. (Fig. 1g) Drag the cart back and forth. It’s alive! Everything should work just as you’d expect.

Now turn off the brake and explore the cart’s behavior. It even bounces a little off the ends of the track.

- Turn on the brake. Drag the largest mass, 200 grams, and drop it when it’s somewhat above the base of the spindle of the cart. It is possible to miss. Just try again.

We now have a cart with total mass 450 g and a mass hanger of mass 50 g. Turn off the brake and observe the system’s motion. You should notice that it moves with less acceleration now with the extra mass. We now have the same mass, 50 g, moving a larger total mass.

I. The Vector Nature of Forces; Free-body diagrams (FBDs)

We’ll start with a puzzle. This will help focus your attention on what can be accomplished by the creative use of Newton’s Second Law. This is also the most complex part of the lab. (You’re welcome.)

With the brake off, 200 g on the cart for a total of 450 g, and the empty mass hanger (50 g) you should see that the cart wants to stay against the right bumper. We now want to investigate the forces acting on this system when we tilt the ramp.
• Move the pointer over the lower section of either end of the track. (Fig. 1f) The pointer will change to a hand or something similar. When this happens you can click and drag up or down to adjust the track angle through a range of ±12°. The cart will respond just like you’d expect.

• Try adjusting the ramp to a specific angle like 5.7°. It’s a bit hard to be that precise. Try this. Click on the left end of the track and with the mouse button still down move your pointer over near the middle of the track. You can still adjust the angle by dragging up and down, but not very precisely. With the mouse still down, drag way out to the left past the end of the track. You can still adjust the track angle, but with much greater precision.

• Now, using this new skill, and with the same masses try to find an angle where the cart will remain motionless. Note that you have to over-tilt to slow it down, and then reduce the tilt when it’s about stopped. You can also play with the brake.

• Give up? You can’t exactly make it stop with these specific masses since the angle is only adjustable to within .1°. But you can get close using the brake to calm things down.

Turn on the brake and set the track angle to 6.4°. Drag the cart to the middle and release. It just barely accelerates down the track. So 6.4° is a bit steep. Now try 6.3°. Now it just barely accelerates up the track.

So we can’t find the angle experimentally, but with our recently-developed mathematical skills we should be able to find just what the balancing angle is.

1. Based on this information, state a possible value for the balancing angle, θ = ________°

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**Figure 2**  
(The vertical section of string has been shortened to reduce the size of the figure.)

**A. Forces on the mass hanger**

In Figure 3 we illustrate the forces acting on our mass hanger. In Figure 3a we arbitrarily pick the +x direction for the hanger as downward. (This will seem more reasonable when we connect it to the cart.) Then in Figure 3b we see a mass hanger free-falling downward at g as it would if there were no string attached. The net force is due entirely to the force of gravity on the hanger, the hanger’s weight. The resulting acceleration is “g” as expected. Make sure you understand the math in each figure. These forces and accelerations are all vectors. And we’re using vector addition. That’s where the signs come from.

You can actually see this freefall by clicking on the Zombie Cart icon. It and the actual cart will become translucent zombie carts to indicate that the cart has become massless. After you try this, click the icon again to bring the cart back to life. Reattach the 200 g mass to the cart.

In Figure 3c a string with upward tension T will change the net force and as a result, change aₓ. There are three possible resulting accelerations. (You’ll understand why we were so careful with the sign of the acceleration in the acceleration lab.)

- If T > W, the net force and acceleration will be upward (negative).
- If T < W, the net force and acceleration will be downward (positive).
- If T = W, the net force and acceleration will be zero, thus maintaining its (constant) velocity or state of rest.
B. Forces on the cart

The forces on the cart are more complex since the downward force of gravity acts at an angle to the track thus acting somewhat down the ramp and somewhat into the track. For this reason we need to consider forces both parallel to the ramp and perpendicular to it. The parallel forces determine the acceleration of the cart along the track, while the perpendicular forces determine the amount of friction between the cart and the track.

Keep the masses on the cart and hanger as they were in Section 1, but set the track angle to 0°. Turn on the brake and move the cart over near the left end of the track. In Figure 4a we illustrate the three forces acting on our cart on a level track assuming no friction. (There’s an extra copy of Figure 4 at the end of the lab for your convenience.)

To see a real time representation of all these vectors, turn on Dynamic Vectors by clicking on the icon. At the center of the track you see a collection of vectors of various colors. Some of them have zero magnitude but their names are still displayed. On the left side of the screen there is a legend explaining the colors and lengths of the vectors.

Drag the cart from side to side to see the velocity vector change in magnitude and direction. Also note that Figure 3c is displayed above the weight hanger. (r: right side) Turn the brake off and on to see everything changing with the situation.

Tilt the track up and down to see all the components.

Note: When Dynamic Vectors are turned on and the cart is at the center of the track it’s hard to attach strings to it. Just move the cart to the side when necessary.
Now back to our investigation. Reset $\theta$ to 0°. The cart’s weight acts downward, currently perpendicular to the track. The resulting equal magnitude normal force acts upward. We pick the $+y$ direction to be perpendicular to the ramp and upward. This is convenient since the normal force always acts in that direction.

We pick the $+x$ direction for the cart parallel to the ramp and to the right. This is also a convenient choice of axis since the tension will be along that axis as will the cart’s motion.

When we tilt our ramp (try it) at an angle $\theta$ in Figure 4b the $x$ and $y$-axes also rotate through the angle $\theta$ and we now have an angle $\theta$ between the $y$-axis and the weight vector which is still downward. The $x$-axis and tension remain parallel to the ramp.

In Figure 4c we include all the forces and components of interest. The weight vector is resolved into its $x$ and $y$ components. Note that in the lab apparatus the orange Normal, and the light pink $W_x$, and $W_y$ vectors are drawn half-size. There’s a darker pink second $W(x)$ on the track which is drawn full size.

- The weight vector’s $x$-component, $W_x$, equals $mg \sin(\theta)$ and represents the extent to which the weight vector acts parallel to the ramp. It acts in the $-x$ direction.
  - Opposing $W_x$ is the tension force acting in the $+x$ direction. (Plus $F(f)$ from the brake if it’s on.)
  - Any friction forces would also act along this axis. Their ($\pm$) direction depends on the situation. (This can be a major source of difficulty for most students. Be sure to revisit this apparatus for clarification when you need it.)

- The weight vector’s $y$-component, $W_y$, equals $mg \cos(\theta)$ and represents the extent to which the weight vector acts perpendicular to the ramp. It acts in the $-y$ direction. As we’ve observed, our cart doesn’t accelerate along the $y$-axis so it must have an equal and opposite force acting on it in the $+y$ direction. This is the normal force acting on the cart. It determines the amount of friction acting. ($F_f \leq \mu F_N$ depending on what other forces are present.)

Similar to the mass hanger, the cart’s motion is determined by the relative values of $T$ and $W_x$. There are three possible resulting accelerations. (Careful! Accelerations, not velocities.) Again, use the dynamic vectors to clarify this. With the brake off, observe the $W_x$, $T$, and $F_{net}$ vectors at about 5° and at 8°. Throw the cart up or down the ramp as needed. Watch $F_{net}$ as you adjust the ramp angle up and down between those angles. You should observe the following:

- If $T > W_x$, the net force and acceleration will be up the ramp (positive).
- If $T < W_x$, the net force and acceleration will be down the ramp (negative).
- If $T = W_x$, the net force and acceleration will be zero, thus maintaining a constant velocity or state of rest.

Before you continue, be sure to verify that the velocity’s direction is immaterial. For each case throw the cart and watch the velocity change direction as the car recoils off the ends. Note that the other vectors are unaffected. (You can’t really verify this for the third case since we’ve seen that we can’t hold the cart still by adjusting the angle, but you should get the idea.)

**C. Forces on our two-mass system**

Where were we? Oh yeah, we said

“So we can’t find the angle (where the cart will remain at rest) experimentally, but with our recently-developed mathematical skills we should be able to find just what the balancing angle is.”

Because our two-mass system is joined by a taught, massless string, the whole system moves with a common acceleration. The direction of this acceleration is different for each mass because the pulley redirects the tension force in the string. We’ve eliminated that problem by our selection of an $x$-axis “with a bend in it.”

![Figure 5](image-url)
If you step back and have a look at the system of one cart, one hanger, and a connecting string, you can see that the forces that actually accelerate the system, that is the external forces, are the weight of the hanger $W_h$, and the x-component of the weight of the cart, $W_{cx}$. The tension, $T$, is actually an interior force within the system. We can write the second law for this two-mass system by combining equations we wrote for each part of the system. Note that the tension force is an external force for each of these parts, but an internal force to the whole system. Writing $\sum \vec{F}_x = m\vec{a}_x$ for each part of the system we find:

For our mass hanger we have $$W_h - T = m_h a_x$$
For our cart on the ramp we have $$T - W_{cx} = m_c a_x$$

We can eliminate the tension by adding our two equations. We’ll add the left sides together and the right sides together.

For the two-mass system we have $$W_h - W_{cx} = (m_h + m_c) a_x$$ Equation 1

Equation 1 looks just right. It says that the net external force equals the total mass of the system times its acceleration.

In our particular quest to find the angle where the system will sit still or move at a constant speed, we can go one step further since for our equilibrium situation, $a_x = 0$. Thus,

$$W_h - W_{cx} = (m_h + m_c) a_x = 0$$

so

$$W_h = W_{cx}$$

From our diagrams we know that for the hanger $$W_h = m_h g$$
And for the cart $$W_{cx} = m_c g \sin(\theta)$$

So, equating $W_h$ and $W_{cx}$,

$$m_h g = m_c g \sin(\theta)$$

1. Let’s try it. With a 50-g hanger and a 450-g cart, the angle for equilibrium, $\theta = \ldots^\circ$

Show calculations here.

So now we see why our cart accelerated in one direction at 6.3° and in the other at 6.4°. This $\sum \vec{F} = m\vec{a}$ equation is a pretty powerful tool.

2. Here’s an interesting follow-up question to debate with your lab partner. If you added a further 50 grams to each member of the system – the cart and the hanger – would the equilibrium angle (when $a = 0$) increase or decrease? Explain your reasoning. Think about relative effect of adding 50 grams to each mass.

3. Calculate that new equilibrium angle for this arrangement. $\ldots^\circ$

Show calculations here.

4. Try it. If your results don’t match your prediction, maybe you need to check your figures.
**II. Accelerated system of a cart on a level track with mass hanger and no friction**

In this part of the lab you’ll be working with a much simpler arrangement - a level track with no friction. It would probably help to leave the dynamic vectors on. By now you should need less explanation of how the apparatus works as well as how we develop the mathematics.

1. Set up your system with 200 grams on the cart and an empty mass hanger on the right side. (Figure 6.)

![Figure 6](image)

![Figure 7](image)

**Important:** In all the vector diagrams you’ll draw, use a consistent scale for the force vectors as in Figure 7.

In figure 8a you see a drawing of a mass hanger in free fall along with a corresponding free-body diagram.

In figure 8b draw a drawing and a FBD for your mass hanger when it’s connected to the cart and the brake is on. You’ll just draw 1) the cord and mass hanger, and 2) T and $W_h$ vectors. Look back at earlier figures and at the Dynamic Vectors on your lab apparatus for help. The apparatus will show you all these vectors.

<table>
<thead>
<tr>
<th>8a. Drawing and FBD for <strong>hanger</strong> in free fall.</th>
<th>8b. Drawing and FBD for <strong>hanger</strong> with brake on.</th>
<th>8c. FBD for <strong>hanger</strong> with brake off.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="8a. Drawing and FBD for hanger in free fall." /></td>
<td><img src="image" alt="8b. Drawing and FBD for hanger with brake on." /></td>
<td><img src="image" alt="8c. FBD for hanger with brake off." /></td>
</tr>
</tbody>
</table>

Now, turn off the brake and observe what happens to the system (cart and hanger)? Big surprise, huh? Clearly Figure 8b no longer applies. The hanger is falling so… for Figure 8c draw just the FBD for the hanger in this situation. This may call for another debate with your lab partner. Also be sure to turn the brake on and off and watch the weight and tension vectors. But your drawing for 8c is for the brake off situation.

2. Let’s now do the same for the cart. **Remember to keep the same scale as in #1.**

In figure 9b draw the FBD for the cart with the hanger attached and the brake on. Use $F_b$ for the brake’s force and T for tension.

In figure 9c draw the FBD for the cart with the hanger attached and the brake off.

<table>
<thead>
<tr>
<th>9b. FBD for <strong>cart</strong> with mass hanger attached and brake on. (just horizontal forces)</th>
<th>9c. FBD for <strong>cart</strong> with mass hanger attached and brake off. (just horizontal forces)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="9b. FBD for cart with mass hanger attached and brake on." /></td>
<td><img src="image" alt="9c. FBD for cart with mass hanger attached and brake off." /></td>
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</tbody>
</table>

We now have a pair for FBDs (8c and 9c) for our system when the brake is off and it’s allowed to accelerate. Here’s a figure which includes our redirected x-axis and the forces along that axis. Figures 8c and 9c should include only these two forces.
3. Your goal now is to determine the acceleration of the system and the tension in the string. In Section 1 of this lab we worked with a similar, but more complex system which was in equilibrium. Feel free to use it as a guide. Remember, this time we’re not in equilibrium. This time there is an acceleration. Here are a few pointers.

- Each FBD (8c and 9c) is a blueprint for constructing a statement of Newton’s 2nd Law. Start by writing these equations down. Be sure to include subscripts (h for hanger and c for cart) for all mass and weight terms.
- Solve these equations simultaneously to find an equation for the full system just as we did in Section 1c. Use it to calculate the acceleration of the system.

Your answer should look familiar. After all you’re exerting a net force on a system equal one tenth of its weight.

Show calculations here.

\[
a = \text{________ m/s}^2 \\
T = \text{________ N}
\]

Confirm your results experimentally.
Using your virtual apparatus, confirm your results. If you recall, in lab 2.3 you found the acceleration of a cart from the slope of a velocity, time graph. That would be a good method to use here. Look back there for guidance.

4. Sketch the graphs you obtain from Logger Pro.

<table>
<thead>
<tr>
<th>11 a. Position vs. time for cart and hanger</th>
<th>11 a. Velocity vs. time for cart and hanger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>Velocity</td>
</tr>
<tr>
<td>Time</td>
<td>Time</td>
</tr>
</tbody>
</table>
5. Acceleration determined graphically _____ m/s²
6. Compare the theoretical (step 7) and experimental (step 9) values for this acceleration.
   Percentage error ______________ %
   Show calculations for percentage error here.

III. Accelerated system of a cart on a level track with two mass hangers but no friction

It’s a simple extension to add a mass on the left-hand side. Let’s look at it briefly. In Figure 12 we’ve added a second mass hanger. As a result we’ve had to add more subscripts to distinguish the hangers and tensions on the left and right. Note also that the positive direction is “up” on the left, “right” on the track and “down” on the right.

Figure 12

Suppose both hangers are empty and the cart has the usual extra 200 grams on it. So

\[ m_{hL} = 50 \text{ g} \quad m_{hR} = 50 \text{ g} \quad m_c = 450 \text{ g} \]

1. Is this system in equilibrium? Test it with your apparatus.  Yes  No  (Circle one.)

2. \( T_L = \) ________ N \quad \( T_R = \) ________ N

Now, suppose we added 20 grams to the right hanger. The system will certainly accelerate in the +x-direction. Go ahead and try it with your apparatus just to make sure.

3. How do \( T_L \) and \( T_R \) compare now, while the system is accelerating?
   \( T_L > T_R \)  \( T_L < T_R \)  \( T_L = T_R \)  (Circle one.)

Puzzled? There’s more than one way to think about this. Here’s one way. We know that

\[ T_L > .49 \text{ N} \] since it’s able to accelerate the left hanger upward. And
\[ T_R < .69 \text{ N} \] otherwise the right hanger would not accelerate downward.

But that’s not enough to answer the previous question. In Section 2 you calculated the acceleration and tension after deriving equations from Newton’s 2nd law. Maybe we can find the tensions that way. This time we have three masses so we’ll have a 2nd law equation for each of the three masses.

4. Draw the three FBDs for this situation. Be sure to include the subscripts and use a consistent scale. You should find that each of the four forces (\( W_{hL}, T_L, T_R, \) and \( W_fR \)) has a different magnitude, so you’ll need four different lengths.

| 13a. FBD for left hanger | 13b. FBD for cart (just horizontal) | 13c. FBD for right hanger |
From that we can create our three equations.

1) \( T_L - W_{hl} = m_{hl} \cdot a \) left hanger
2) \( T_R - T_L = m_c \cdot a \) cart
3) \( W_{hr} - T_R = m_{hr} \cdot a \) right hanger

Assuming that we know the masses, this gives us three unknowns – \( a \), \( T_L \) and \( T_R \). That shouldn’t be hard to solve. Adding the three equations should eliminate the internal tension forces.

\[
(T_L - W_{hl}) + (T_R - T_L) + (W_{hr} - T_R) = m_{hl} \cdot a + m_c \cdot a + m_{hr} \cdot a
\]

Equation 2

\[
W_{hr} - W_{hl} = (m_{hl} + m_c + m_{hr}) \cdot a
\]

Make sense? All this says is that the net force on the system is the vector sum of the two external forces and that the mass being accelerated is the total mass. You can go straight to this final equation once you catch on the whole system idea.

\[ \Sigma F = ma \]

refers to the external force on and mass of any group of objects.

5. Find the \textbf{acceleration} of this three-mass system and the \textbf{two tensions}.

\textit{Show calculations here.}

\[
a = \underline{\text{m/s}^2}
\]

\[
T_L = \underline{\text{N}} \quad T_R = \underline{\text{N}}
\]

We predicted that \( T_L > .49 \text{ N} \) and \( T_R < .98 \text{ N} \). Hopefully that checks out ok.

\textbf{IV. Atwood’s Machine}

Atwood’s Machine, shown in Figure 14a, is similar to our previous system but simpler. Without the mass in between the two pulleys the tension is the same throughout the string. So we just represent it everywhere as \( T \).

With the cart and two hanger system we just worked with, \( F = ma \) became

\[
W_{hr} - W_{hl} = (m_{hl} + m_c + m_{hr}) \cdot a
\]

Equation 2

Atwood’s machine essentially just removes the cart. Write down \( F = ma \) for each of the two masses and then combine them to find an equation similar to Equation 2.

Show your algebra here. Use the mass and weight notation from Figure 14a.
This would be a good time to note that we are using massless string, which means that its density is so small that it can be neglected in our analysis.

OK, let’s try it in the lab. Oops! We have a problem. All our data collection tools depend on the presence of the cart. So we need the cart to be present but we don’t want its mass to be present. This calls for Zombie Cart‼‼!

<table>
<thead>
<tr>
<th>Figure 14b. Our Version of Atwood’s Machine.</th>
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<tbody>
<tr>
<td>1. Start with system shown in Figure 12. (Section III). (m_L = 50 g, m_R = 70 g). If you then click the Zombie Cart button you’ll just have the two masses. There are two pulleys, but there is nothing in between (except the soulless/massless zombie cart) so it’s equivalent to the Atwood’s machine shown in Figure 14a. Having a second pulley and the string in between adds nothing to the system. It’s just two masses connect by a with a common tension.</td>
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With the real cart present there was a different tension on each side of the cart. Notice how the tension is the same everywhere in Figure 14b. If it weren’t you’d have a tension that changed along the horizontal string. That would be hard to explain.  
... unless you had a string with mass, which we don’t.

2. Calculate the theoretical acceleration using Equation 3.

Show calculations here.

3. Get things set up to take data, start the sensor and then click the zombie button.

Experimental acceleration from Logger Pro __________________________ m/s^2

4. Determine the percentage difference between your experimental and theoretical values for the acceleration.

Show calculations here.
V. Static and Kinetic Friction

You know from experience that when you push a chair you have to push with some minimal amount of force before it slips and begins to move. But after you get it moving, the force necessary to keep it moving at a constant velocity is less than what was required to make it initially slip. In this part of the lab we’ll investigate the nature of these two aspects of friction.

A. Static Friction

If Dynamic Vectors are not on, be sure to turn them on. With the brake off, turn on the friction pad. Set the $\mu_{KS}$ number stepper to zero. Leave it there for the rest of the lab. Put 200 g on the cart. You could think of the brake as a friction pad with a huge friction coefficient. We actually use $\mu_s = 10$ and $\mu_k = 9$. The regular friction pad has more normal static and kinetic friction coefficients. We want to investigate them.

Add a string and mass hanger on the right side.

The cart doesn’t move – it’s still in equilibrium. Note $T_r$ and $F(f)$. The tension pulls it to the right and the static friction force pulls it with equal force to the left. Of course the hanger is also in equilibrium. Add a $F_f$ vector to the first (unlabeled) figure.

1. Draw the FBD’s for the cart and the hanger at equilibrium. Use $F_f$, $T$, and $W_h$ for your labels.

In the steps below you’ll be adding a succession of small masses to the mass hanger. If you’re not certain how many masses are on the hanger, use the right-click Zoom in tool.

2a. What’s the tension in the string? N 2b. What’s the static friction force? N

Add 10 grams to the hanger making it a total of 60 grams counting the hanger. Note the system’s and vectors’ responses.

3a. What’s the tension in the string? N 3b. What’s the static friction force? N

Add 10 grams to the hanger making it a total of 70 grams counting the hanger. Note the system’s and vectors’ responses.

4a. What’s the tension in the string? N 4b. What’s the static friction force? N

Add 10 grams to the hanger making it a total of 80 grams counting the hanger. Note the system’s and vectors’ responses.

5a. What’s the tension in the string? N 5b. What’s the static friction force? N

Add 10 grams to the hanger making it a total of 90 grams counting the hanger. Note the system’s and vectors’ responses.

You’ve just discovered that the maximum amount of static friction is somewhere between .78 N and .88 N.

The static friction coefficient for our cart’s friction pad is programmed to be .18. So the maximum amount of static friction is $F_{static\ max} = \mu_s F_N = \mu_s m_g g = .18 \times .450 \text{ kg} \times 9.8 \text{ N/kg} = .79 \text{ N}$

Just barely in our range.

Hopefully you now understand what we mean when we say that for static friction: $0 < F_{static} \leq F_{static\ max}$

Take this opportunity to study the statement. You might want to write down some thoughts about what it means.

Kinetic friction behaves quite differently. Let have a quick look at it now.
B. Kinetic Friction

You may have been wondering what that “Bump!” button was about. Here’s where we use it. We just verified that static friction varies from zero to some maximum amount, \( F_{\text{static max}} \). If you exert a horizontal force greater than \( F_{\text{static max}} \) the object will accelerate.

\( F_{\text{kinetic}} \) on the other hand is a fixed amount for two objects (and a given normal force.) And it’s always less than \( F_{\text{static max}} \). So,

- When an object is at rest, the friction force, \( F_t \leq F_{\text{static max}} \).
- When an object is moving, the friction force, \( F_t = F_{\text{kinetic}} \).

There is another situation that you’re very familiar with. If you pushed the chair until it began to move you could then back off and push with a smaller (than \( F_{\text{static max}} \)) force and the chair will continue to move as long as your force \( \geq F_k \). That’s what our Bump does. It gives the cart a tiny velocity to break it free; then it can move against the force of kinetic friction.

What we want to do is observe this behavior and see how \( F_{\text{kinetic}} \) responds when we exert varying forces against it.

Put 200 g on the cart. Turn on the friction pad and then turn off the brake. Add a string and empty mass hanger on the right. Move the cart to the left end of the track. (We’ll always start there during these trials.)

The cart doesn’t move. Give it a bump by clicking the “bump” button.

That was interesting. Try it again with the motion sensor on.

After being initially at rest, the cart is given some initial positive velocity. That’s what the bump really does. It gives it a tiny initial velocity which switches it to kinetic friction.

Then in this case it decelerates until it comes to a halt.

What does this mean?

The hanger weighs .49 N. Notice that the cart decelerated immediately after starting to move. Again, watch the vectors. There are two horizontal force vectors plus the net horizontal force shown.

6. Is the kinetic friction force larger or smaller than .49 N?

Larger Smaller (circle one)

You should have observed that it was larger. Click “Bump” a few times and notice how the magnitude (length) of \( F_t \) differs before and after the cart stops moving. Is there something odd going on? While the cart is moving, you have kinetic friction, \( F_k \). Once it stops you have static friction, \( F_s \). It seems that \( F_k \) is greater than \( F_s \). Isn’t it supposed to be the other way around?

Not exactly. \( F_{\text{static max}} > F_{\text{kinetic}} \). But we’re not working with \( F_{\text{static max}} \) here. The friction force, \( F_{\text{static}} \), after it stops moving is given by \( 0 < F_{\text{static}} \leq F_{\text{static max}} \). In this case the static friction force after it stops moving is less than \( F_{\text{static max}} \).

Sorry to get all tied up in this but it’s very important that you see how these two types of friction act.

Suppose you were moving very fast down a gentle hill in your car. You slam on your brakes and skid to halt. During the skid the kinetic friction force was very large. But once you come to a halt, the static friction force would be much smaller. It’s much less than its maximum value. To reach that value you’d have to, say, push the car very hard in a direction down the hill until it just slipped.

How about an opposite sort of example? Could you make a hill steep enough so that you could hold the car at rest just by keeping the brake on, but where you couldn’t stop the car once it got moving? Yup. You could probably easily imagine sitting on such a steep hill where the seat of your pants act as the brake.

OK, we know all the masses and we can find the acceleration from our graph. Let’s see if we can compute \( \mu_k \), the kinetic friction coefficient.
Let’s jump right to the equation. We’ll skip the initial step of writing \( F = ma \) for each mass and then adding the equations. For the entire system we have the hanger’s weight pulling against the friction force and they’re acting on both masses. So

\[
W_h - F_{fr} = (m_c + m_h) a \\
W_h - \mu_k F_N = (m_c + m_h) a \\
m_h g - \mu_k m_c g = (m_c + m_h) a \\
\mu_k m_c g = m_h g - (m_c + m_h) a \\
\mu_k = (m_h g - (m_c + m_h) a)/ m_c g \\
\mu_k = (.05 \text{ kg} \times 9.8 \text{N/kg} - (.45 \text{ kg} + .05 \text{ kg}) a)/(.45 \times 9.8 \text{N/kg})
\]

6. All we need is the acceleration. Use Logger Pro to find it and calculate \( \mu_k \) for the empty hanger case.

**Show calculations here.**

Empty hanger: \( a = \) ____________ m/s\(^2\) \quad \mu_k = \) ____________

7. Repeat for another run with your choice of added mass to the hanger from 10 – 30 grams.

You’ll note this time that \( F_{static} > F_{kinetic} \).

**Show calculations here.**

Hanger with ______ grams added: \( \mu_k = \) ______________

8. Was the sign of the acceleration the same in both cases? What does this indicate about the forces in each case? Explain below.

10. We said earlier that: \( 0 < F_{static} \leq F_{static max} \)

Does the kinetic friction vary in a similar way? \( \quad \) Yes \quad No \quad (circle one)
Figure 4

\[ \Sigma F_x = T - W_x = mg = ma_x \]

\[ \Sigma F_y = F_N - W_y = ma_y = 0 \]

\[ a_x = \frac{T - W_x}{m} \]

\[ F_N = W_y \]